
UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2008/2009

April 2009

JIF 318 – Quantum Mechanics
[JIF 318 – Mekanik Kuantum]

Time : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question carries 100 marks.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

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Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

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1. (a) What is Black Body Radiation? Why did the classical physics fail to explain the properties of Black Body Radiation?
(30 marks)
- (b) How did Planck overcome the problems in explaining the experimental properties of the Black Body radiation?
(30 marks)
- (c) Describe two experimental evidences that indicate the wave properties of electron.
(40 marks)
2. (a) What is the characteristic of a linear operator?
(10 marks)
- (b) Show that the sum and product of two linear operators are linear operators.
(10 marks)
- (c) Calculate $\left(\frac{1+i}{1-i}\right)^5$ using complex number Cartesian coordinate and polar coordinate.
(20 marks)
- (d) If the wave function of a wave travelling in one dimension in the positive x direction is given by

$$\Psi = A \cos(2\pi x / \lambda - 2\pi \nu t),$$

show that the Schrödinger equation can be given as

$$\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}.$$

(60 marks)

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3. (a) One dimensional Schrödinger equation for a infinite square well potential is given by

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

Find the solution of this equation if the potential $V(x)$ is given by

$$V(x) = 0 \text{ for } 0 \leq x \leq a$$

$$V(x) = \infty \text{ for } x < 0 \text{ or } x > a$$

where a is the width of the square well potential.

(80 marks)

- (b) Prove that the mean value for the position of the particle in the infinite square well is

$$\langle x \rangle = \frac{a}{2}$$

(20 marks)

4. The Schrödinger equation of a bound state for potential $V(x) = 0$ for $0 \leq x \leq a$ and $V(x) = \infty$ for $x < 0$ or $x > a$ is

$$\frac{d^2 \psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0$$

By taking into consideration the appropriate boundary conditions, show that

- (a) the wave function is given by

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

(40 markah)

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- (b) the energy of the bound state E is given by

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

where $n = 1, 2, 3, \dots$

(30 marks)

Plot $\psi_n(x)^* \psi_n(x)$ for $n = 1, 2, 3$. Describe the physical interpretation of these plots.

(30 marks)

5. (a) If one dimensional Hamiltonian operator H is $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$ and one- dimensional momentum operator P is $-i\hbar \frac{d\psi}{dx}$, show that

$$[H, P] = i\hbar \frac{\partial V}{\partial x}$$

(40 marks)

- (b) Explain the following:

- (i) Vector space
- (ii) Inner product
- (iii) Adjoint Operator
- (iv) Hermitian operator
- (v) Basis set
- (vi) Commutator
- (vii) Orthonormal Basis

(60 marks)

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1. (a) *Apakah Sinaran Jasad Hitam? Kenapa fizik klasikal menjelaskan sifat-sifat sinaran hitam?*
(30 markah)
- (b) *Bagaimana Planck mengatasi masalah dalam menjelaskan sifat-sifat ujikaji Sinaran Jasad Hitam?*
(30 markah)
- (c) *Jelaskan dua bukti ujikaji yang menunjukkan sifat gelombang elektron.*
(40 markah)
2. (a) *Apakah ciri operator linear?*
(10 markah)
- (b) *Tunjuk bahawa hasil campur dan hasil darab dua operator linear ialah operator linear.*
(10 markah)
- (c) *Kira $\left(\frac{1+i}{1-i}\right)^5$ menggunakan nombor kompleks koordinat cartesian dan koordinat pola.*
(20 markah)
- (d) *Jika fungsi gelombang suatu gelombang bergerak dalam satu dimensi dalam arah x positif x diberikan oleh*

$$\Psi = A \cos(2\pi x / \lambda - 2\pi vt),$$

tunjukkan persamaan Schrödinger boleh diberikan oleh

$$\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}.$$

(60 markah)

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3. (a) *Persamaan Schrödinger satu dimensi untuk satu telaga keupayaan segi empat tak terhingga diberikan oleh*

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

Cari penyelesaian persamaan ini jika keupayaan $V(x)$ diberikan oleh

$$V(x) = 0 \text{ for } 0 \leq x \leq a$$

$$V(x) = \infty \text{ for } x < 0 \text{ or } x > a$$

di mana a ialah kelebaran keupayaan segiempat.

(80 markah)

- (b) *Buktikan nilai $\langle x \rangle$ untuk kedudukan zarah dalam telaga segiempat tak terhingga ialah*

$$\langle x \rangle = \frac{a}{2}$$

(20 markah)

4. *Persamaan Schrödinger untuk keadaan terikat bagi keupayaan $V(x) = 0$ untuk $0 \leq x \leq a$ dan $V(x) = \infty$ untuk $x < 0$ atau $x > a$ ialah*

$$\frac{d^2 \psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0$$

Dengan mengambil kira syarat-syarat batasan yang sesuai, tunjukkan bahawa

- (a) *fungsi gelombang diberikan oleh*

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

(40 markah)

...7/-

- (b) Tenaga keadaan terikat E diberikan oleh

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

di mana $n = 1, 2, 3, \dots$

(30 marks)

Lakarkan $\psi_n(x) * \psi_n(x)$ untuk $n = 1, 2, 3$. Jelaskan interpretasi fizikal lakaran-lakaran ini.

(30 markah)

5. (a) Jika operator Hamiltonian satu dimensi H ialah $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$ dan operator momentum satu dimensi, P ialah $-i\hbar \frac{d\psi}{dx}$, tunjukkan

$$[H, P] = i\hbar \frac{\partial V}{\partial x}$$

(40 markah)

- (b) Jelaskan perkara-perkara berikut:

- (i) Ruang vektor
- (ii) Hasil darab berkedalaman
- (iii) Operator Adjoint
- (iv) Operator Hermitian
- (v) Set asas
- (vi) Kommutator
- (vii) Asas Orthonormal

(60 markah)